DESIGN OF COMMENSURATE TRANSMISSION LINE CIRCUITS*

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Abstract

Digital computer techniques are developed for the approximation of the periodic frequency characteristics of commensurate transmission line circuits. For a given periodic delay or loss-derivative function the system function is derived automatically using a direct method. Any suitable synthesis program will complete the design.

Introduction

As a basis for the development of this technique the example of the periodic delay function has been used, leading to a transmission line circuit function which may be realized by a suitable synthesis method.[1] It is stressed however that the method applies generally to either delay or loss derivative curves. An initial approximation is realized by entirely linear techniques, not requiring iterative methods of solution. This compares favorably with approximation techniques used for lumped constant element circuits that require iterative solutions to a number of nonlinear equations.[2,3]

Following the initial approximation an optimization program may be needed to minimize the error. For the delay function application used the error is minimized by increasing the order of the system function and subsequent deletion of cascaded line elements with impedances close to the load impedance ($R_L = 1$). This method is dependent however upon the cascade synthesis method and since it is expected that other synthesis methods will be developed this method of optimization will not be generally applicable.

The Approximation Method

It is feasible to write a Fourier Series approximation for a given periodic function and to obtain a set of n linear equations relating the Fourier Series coefficients to the coefficients of the series resulting from

*This work was supported by the National Science Foundation Grant GK-1545

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NOTES

division of the part of the (unknown) system function representing delay or of the part representing loss-derivative.

A given Fourier series representing the delay function may be transformed, term by term into a "loss-derivative" function, by means of the Hilbert transform. The given series:

$$\frac{\tau(\omega)}{T} = \tau_o + \tau_2 \cos 2\,\omega\,T + \tau_4 \cos 4\,\omega\,T + \ldots + \tau_n \cos n\,\omega\,T + \ldots \qquad (1)$$

$$\frac{1}{T}\frac{dG(\omega)}{d\omega} = - \tau_2 \sin 2\,\omega\,T - \tau_4 \sin 4\,\omega\,T - \ldots - \tau_n \sin n\,\omega\,T + \ldots \qquad (2)$$

Combining these series into one:

$$\frac{1}{T}\left(\tau(\omega) + j\,\frac{dG}{d\omega}\right) = \left[\tau_o + \tau_2\, e^{-2sT} + \tau_4\, e^{-4sT} + \ldots + \tau_n\, e^{-nsT}\right]_{s=j\omega} \qquad (3)$$

Consider the function:

$$F(s) = \frac{1}{Q(s)} = \frac{1}{\left[\alpha_n \tanh^n sT + \alpha_{n-1} \tanh^{n-1} sT + \ldots + \alpha_o\right]\cosh^n sT} \qquad (4)$$

To separate (4) into loss-derivative and delay function[4] write:

$$\left[-\frac{F'(s)}{F(s)}\right]_{s=j\omega} = \left[j\,\frac{d}{d\omega}\ln F(s)\right]_{s=j\omega} =$$

$$= j\,\frac{d}{d\omega}\left[\ln\left|F(j\omega)\right| + j\,\theta(\omega)\right] = j\,\frac{dG(\omega)}{d\omega} + \tau(\omega) \qquad (5)$$

To obtain (5) from (4) the latter equation is rewritten, using

$$\tanh sT = \frac{e^{2sT} - 1}{e^{2sT} + 1} \qquad (6)$$

which results in:

$$F(s) = \frac{2^n . e^{nsT}}{\sum_{k=0}^{n} a_k (e^{2sT}-1)^k\,(e^{2sT}+1)^{n-k}} \qquad (7)$$

or

$$F(s) = \frac{2^{n}.e^{nsT}}{\sum_{k=0}^{n} A_k \, e^{2ksT}} = \frac{R(s)}{S(s)} \qquad (8)$$

Assuming that the delay function is to be used in the approximation process, consider

$$\tau(\omega) = \text{Re}\left[-\frac{F'(s)}{F(s)}\right]_{s=j\omega} = -\text{Re}\left[\frac{F'(j\omega)}{F(j\omega)}\right] \qquad (9)$$

From (8)

$$-\tau(\omega) = \text{Re}\left[\frac{R'(s)}{R(s)} - \frac{S'(s)}{S(s)}\right]_{s=j\omega}$$

or

$$-\tau(\omega) = \frac{1}{2}\left[\left(\frac{R'(s)}{R(s)} + \frac{R'(-s)}{R(-s)}\right) - \left(\frac{S'(s)}{S(s)} + \frac{S'(-s)}{S(-s)}\right)\right]_{s=j\omega} \qquad (10)$$

It can be shown that:

$$\frac{1}{2}\left[\frac{R'(s)}{R(s)} + \frac{R'(-s)}{R(-s)}\right]_{s=j\omega} = nT \qquad (11)$$

After long division of $\frac{S'(s)}{S(s)}$ and $\frac{S'(-s)}{S(-s)}$ respectively, where $S(s) = \sum_{k=0}^{n} A_k \, e^{2ksT}$, and addition we find:

$$\frac{1}{2}\left[\frac{S'(s)}{S(s)} + \frac{S'(-s)}{S(-s)}\right] = 2nT + \frac{T}{2}\Big[\tau_2(e^{2sT}+e^{-2sT}) + \tau_4(e^{4sT}+e^{-4sT}) + \ldots$$
$$\ldots + \tau_{2n}(e^{2nsT} + e^{-2nsT})\Big] \qquad (12)$$

Combining (11) and (12):

$$\tau(\omega) = \mathrm{Re}\left[-\frac{F'(s)}{F(s)}\right]_{s=j\omega} =$$

$$= nT + T\left[\tau_2 \cos 2\omega T + \tau_4 \cos 4\omega T + \ldots + \tau_{2n} \cos 2n\omega T\right] \qquad (13)$$

As the division process used to obtain (12) may be expressed in a series of linear equations, the process of obtaining the polynomial coefficients α_k in (4) from given τ_k in (1) or (2) is linear also. It consists of two steps:

1) obtain the coefficients A_k from given τ_k
2) obtain the coefficients α_k from A_k

Both steps may be formulated in matrix form and may easily be solved using a digital computer.

Implementation

As no effort was expanded to minimize the error in the approximation process, the results shown are initial approximations only. This however is felt to be a major contribution, as such a method appears to be unavailable.

To implement the results for delay functions, the function F(s) in (4) should be interpreted as the transfer impedance of a cascade of transmission lines. Alternatively the function F(s) may be interpreted as the transmission coefficient of a cascade of coupled line all-pass sections, the synthesis is identical to the one for the cascaded lines however.[1]

The synthesis techniques for cascaded lines often leads to a set of line impedance values of wide range. Methods of limiting this range, by using either combinations of low-order cascades or other synthesis methods, are the subject of further studies.

The other disadvantage, that of finding impedance values $Z_i < 1$ that cannot be realized in coupled line form, is encountered only for certain types of functions, such as linear delay with negative slope and for discontinuous functions.

Several results are shown here, indicating both the character of the approximation for several types of periodic delay functions and the resulting impedances for the cascaded line values. Further studies are required to obtain realizability and, for the cascade synthesis method, impedance values within range for practical realization.

References

1. W. Steenaart, "A Contribution to the Synthesis of Distributed All-Pass Networks," Proc. of the Symposium on Generalized Networks, Poly. Inst. of Brooklyn, New York, April 1966, pp. 173-192.

2. J. V. Fall, "A Digital Computer Program for the Design Phase Correctors," Trans. IRE, vol. CT-8, pp. 223-236, Sept. 1961.

3. S. Hellerstein, "Synthesis of All-Pass Delay Equations," ibid., pp. 215-222.

4. G. G. Gouriet, "Two Theorems Concerning Group-Delay with Practical Application to Delay Correction," Proc. I.E.E., vol. 105, Part C, pp. 240-244, 1958.

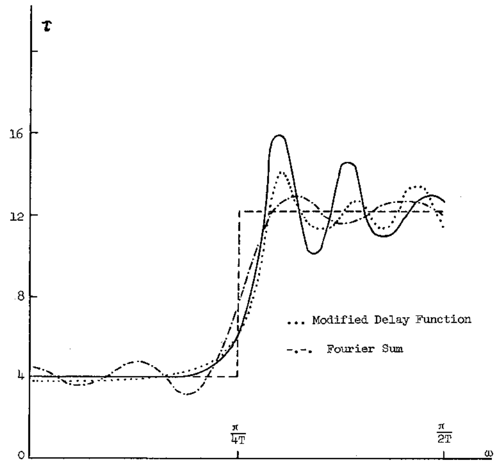

FIG. 1 - CONSTANT DELAY APPROXIMATION OVER 50% BANDWIDTH. DELAY MAGNITUDE 8 SECONDS. $N = 8 = 2N_{min}$.

$Z_1 = 2687.220$	$Z_5 = 1.679$
$Z_2 = 216.797$	$Z_6 = .868$
$Z_3 = 29.556$	$Z_7 = .932$
$Z_4 = 5.597$	$Z_8 = 1.081$

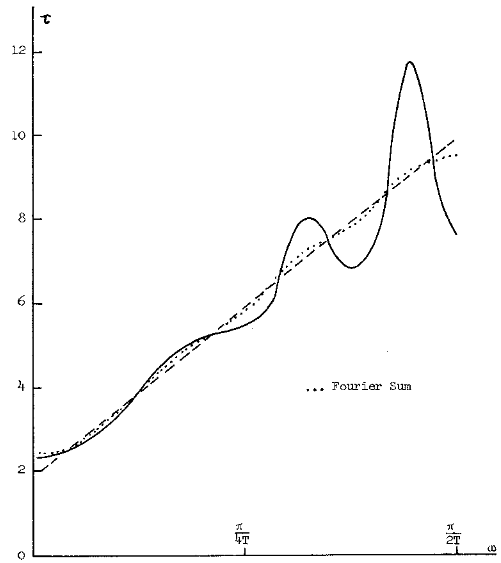

FIG. 2 - LINEAR DELAY SLOPE APPROXIMATION.
SLOPE = 8 SECONDS, N = 6 = 1.5 N_{min}.

Z_1 = 179.273 Z_4 = 1.967
Z_2 = 15.640 Z_5 = 1.295
Z_3 = 3.993 Z_6 = 1.199

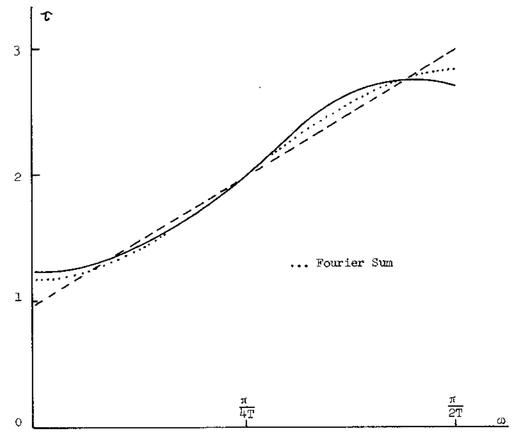

FIG. 3 - LINEAR DELAY SLOPE APPROXIMATION.
SLOPE = 2 SECONDSλ N = 2 = $2N_{min}$.

Z_1 = 2.59
Z_2 = 1.18

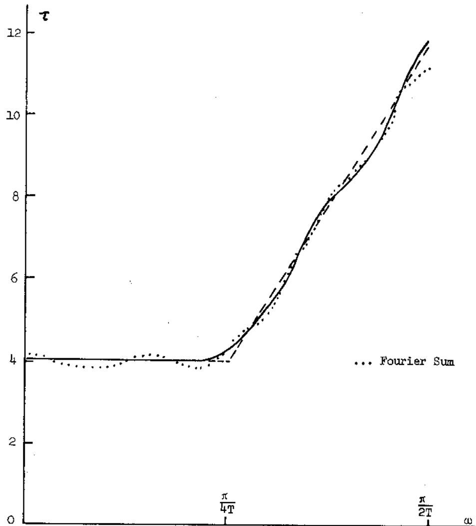

FIG. 4 - LINEAR DELAY SLOPE APPROXIMATION OVER 50% BANDWIDTH.
SLOPE = 8 SECONDS, N = 6 = $3N_{min}$.

$Z_1 = 132.797$ $Z_4 = 0.903$
$Z_2 = 6.436$ $Z_5 = 0.992$
$Z_3 = 1.373$ $Z_6 = 1.033$

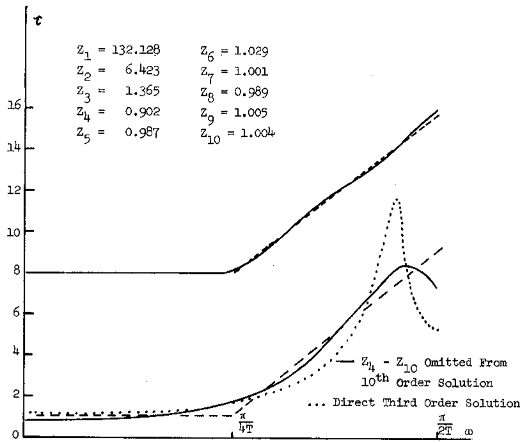

FIG. 5 - LINEAR DELAY SLOPE APPROXIMATION OVER 50% BANDWIDTH.
SLOPE = 8 SECONDS, N = 10 = $5N_{min}$.